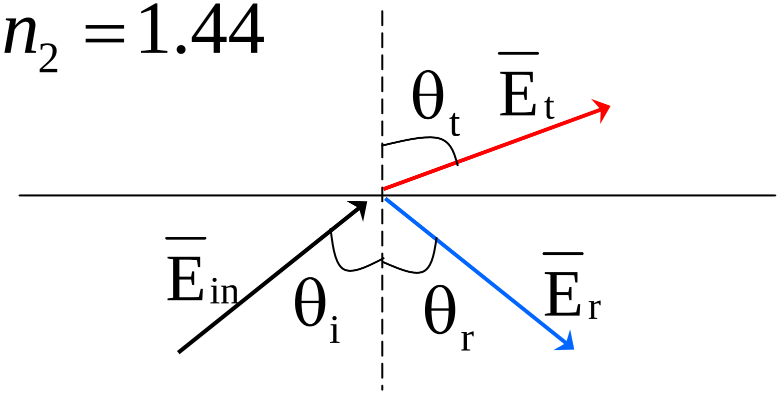
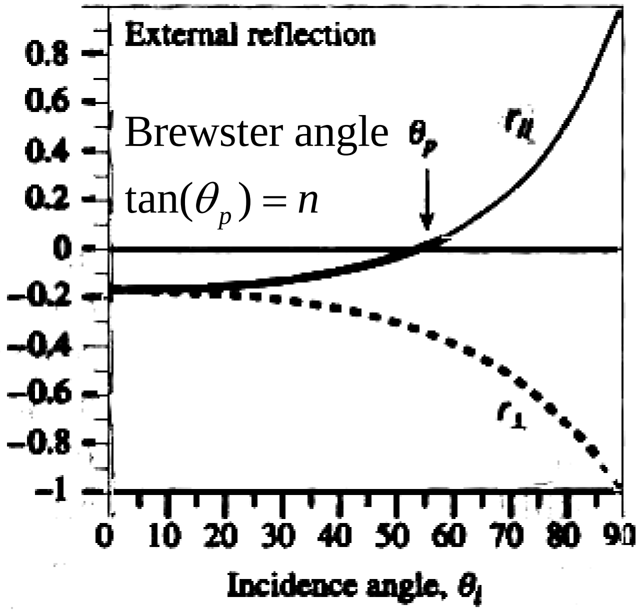
Lect. 4: Total Internal Reflection

$n_2 = 1.44$



$n_1 = 1.0$



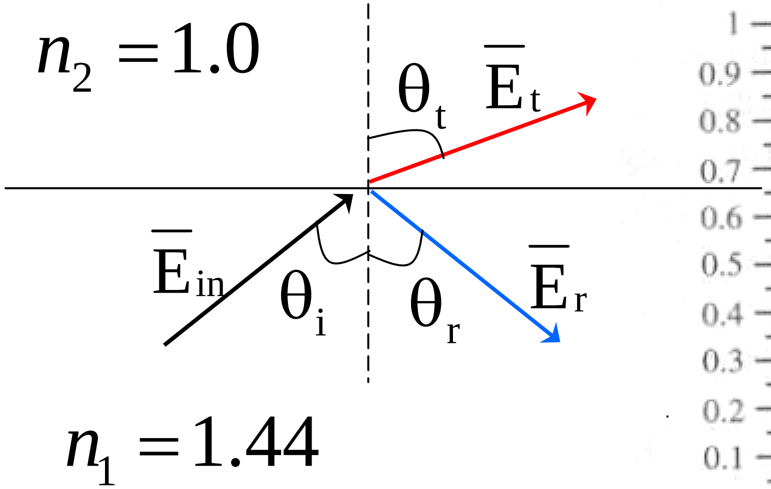
$$r_\perp = \frac{E_r}{E_i} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

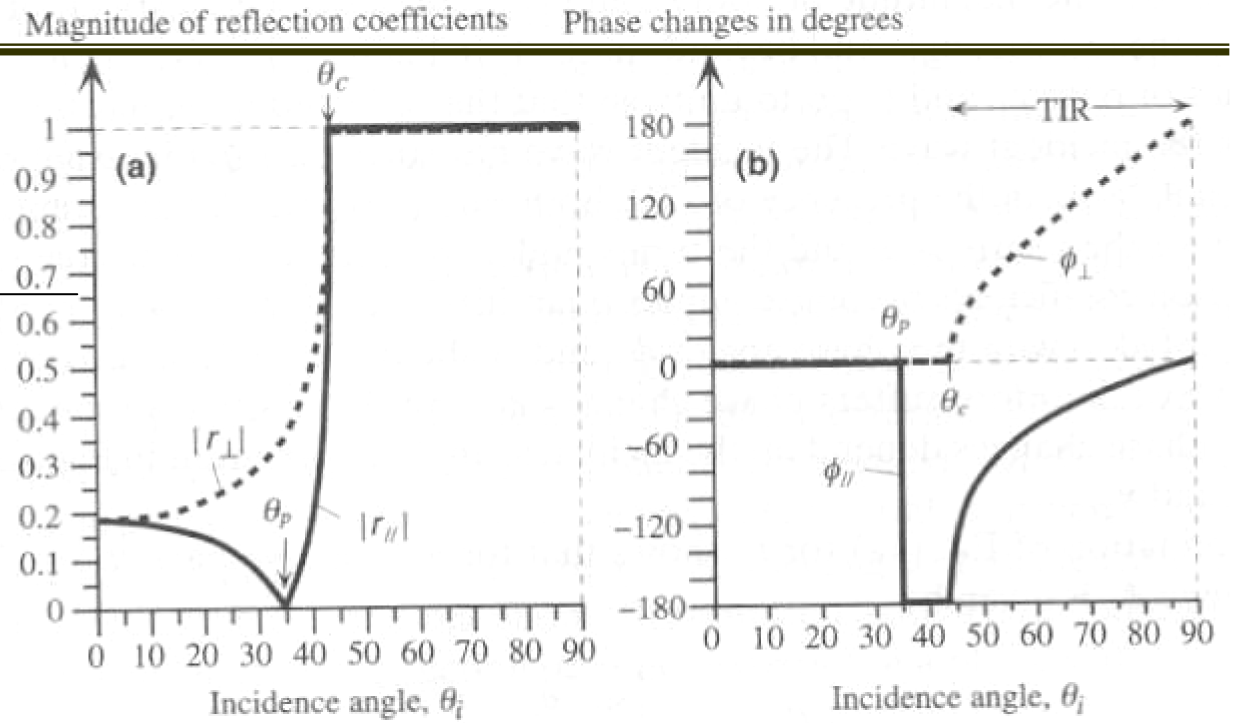
$$r_\parallel = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_\perp = \frac{E_t}{E_i} = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}} \qquad (n = \frac{n_2}{n_1})$$

$$t_\parallel = \frac{2n\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

Lect. 4: Total Internal Reflection





$$r_\perp = \frac{E_r}{E_i} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$r_\parallel = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_\perp = \frac{E_t}{E_i} = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}} \qquad \left(n = \frac{n_2}{n_1}\right)$$

$$t_\parallel = \frac{2n\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

Lect. 4: Total Internal Reflection

From $r_\perp = \dfrac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$

$\sin\theta_i > n \quad (\because\ \sin\theta_i > \sin\theta_c = n)$

Let $\left[n^2 - \sin^2\theta_i\right]^{1/2} = -j\left[\sin^2\theta_i - n^2\right]^{1/2}$

$E_i + E_r = E_t$

$\Rightarrow t_\perp = 1 + r_\perp$

$$r_\perp = \frac{\cos\theta_i + j\left[\sin^2\theta_i - n^2\right]^{1/2}}{\cos\theta_i - j\left[\sin^2\theta_i - n^2\right]^{1/2}} = |r_\perp| e^{j\phi_\perp}$$

$$|r_\perp| = 1 \text{ and } \phi_\perp = \tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right) - \left(-\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right)\right) = 2\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i}\right)$$

$$\left(\tan\left(\frac{\phi_\perp}{2}\right) = \frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i} \right)$$

Lect. 4: Total Internal Reflection

From $r_{\parallel} = \dfrac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$

$$H_i + H_r = H_t$$

$$\Rightarrow 1 + \frac{H_r}{H_i} = \frac{H_t}{H_i}$$

$$1 - r_{\parallel} = \frac{E_t/\eta_2}{E_i/\eta_1} = nt_{\parallel}$$

$\sin\theta_i > n \quad (\because \ \sin\theta_i > \sin\theta_c = n)$

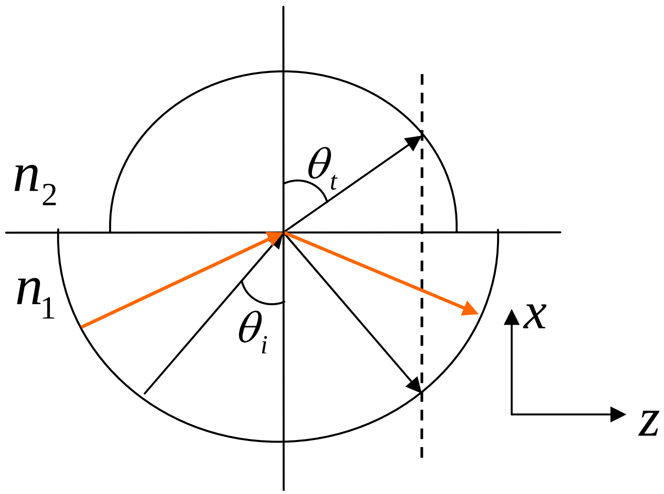
Let $\left[n^2 - \sin^2\theta_i\right]^{1/2} = -j\left[\sin^2\theta_i - n^2\right]^{1/2}$

$$r_{\parallel} = \frac{-j\left[\sin^2\theta_i - n^2\right]^{1/2} - n^2\cos\theta_i}{-j\left[\sin^2\theta_i - n^2\right]^{1/2} + n^2\cos\theta_i} = \left|r_{\parallel}\right| e^{j\phi_{\parallel}}$$

$$\left|r_{\parallel}\right| = 1 \text{ and } \phi_{\parallel} = -\pi + \tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i}\right) - \left(-\tan^{-1}\left(\frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i}\right)\right)$$

$$\tan\left(\frac{\phi_{\parallel} + \pi}{2}\right) = \frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i}$$

Lect. 4: Total Internal Reflection



$n_1 \sin\theta_i = n_2 \sin\theta_t$

If $n_1 > n_2$, $\theta_t > \theta_i$.

With n_1 increasing, θ_t reaches 90° beyond which no tranmission is possible

$$\theta_c \text{ (critical angle)} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$k_z = k_{t,z} = k_{r,z}$$

$$n_1 k_0 \sin\theta_i = n_2 k_0 \sin\theta_t$$

$$k_t^{\ 2} = k_{t,z}^2 + k_{t,x}^2$$

$$when \quad k_{t,z}^{\ \ 2} > k_t^{\ 2}$$

$$k_{t,x}^2 < 0 \rightarrow k_{t,x} = -j\alpha$$

$$\therefore E_t = tE_i e^{(-\alpha x)} e^{(-jk_{t,z}z)}$$

$$k_{t,x}^2 = k_t^{\ 2} - k_{t,z}^2 = (n_2 k_0)^2 - (n_1 k_0 \sin\theta_i)^2$$

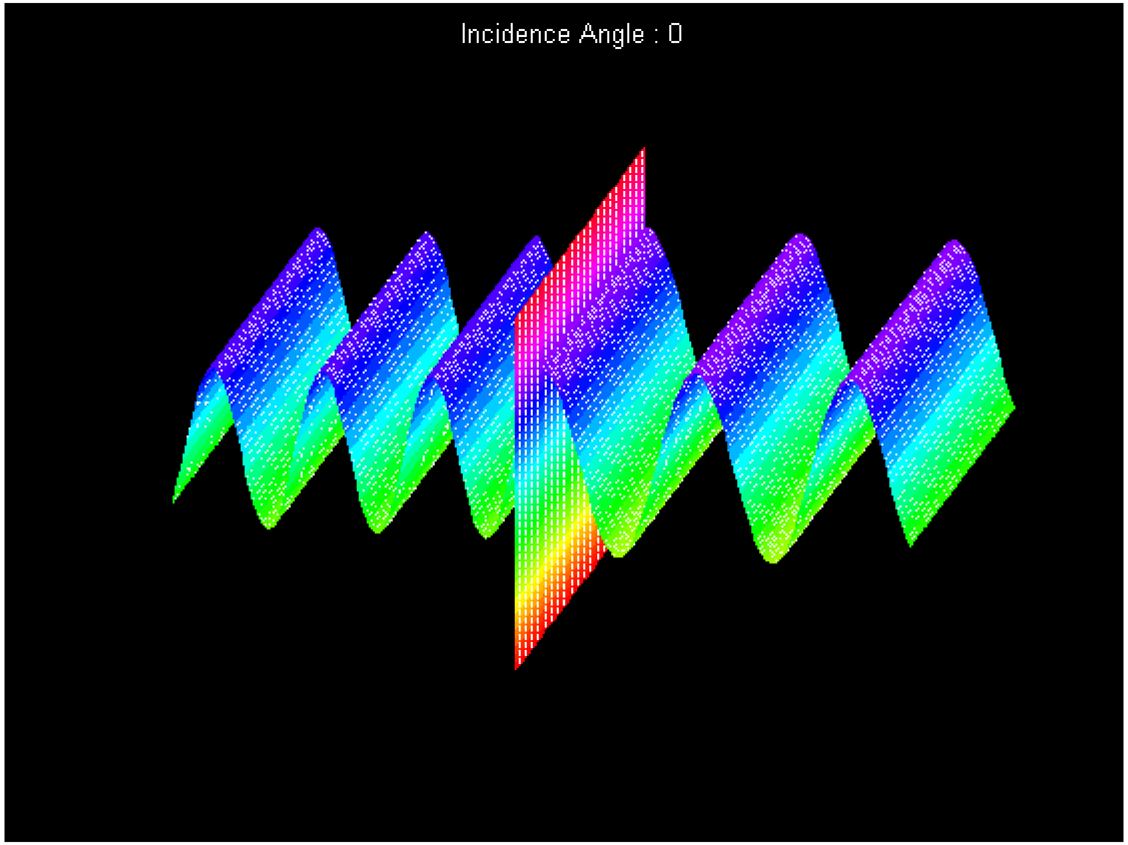
$$k_{t,x} = -j\alpha = -j[(n_1 k_0 \sin\theta_i)^2 - (n_2 k_0)^2]^{\frac{1}{2}}$$

$$\therefore \alpha = k_0[(n_1 \sin\theta_i)^2 - n_2^{\ 2}]^{\frac{1}{2}}$$

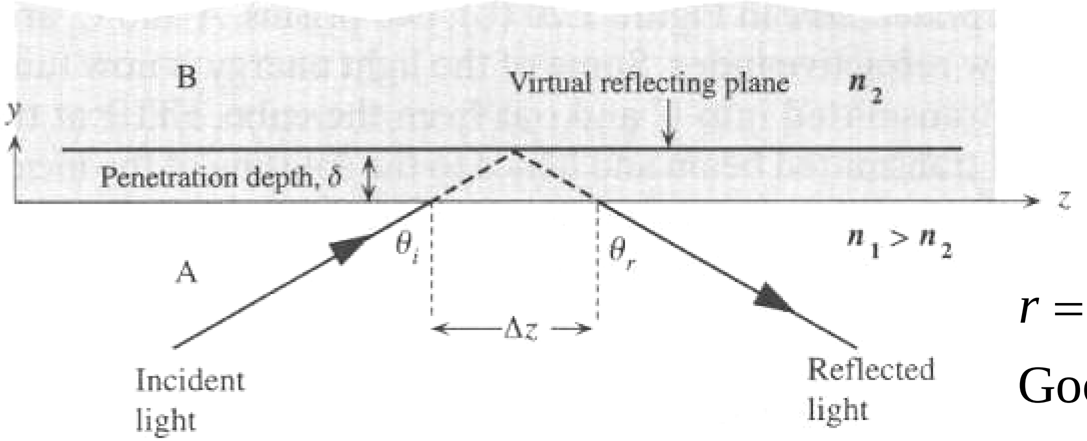
Lect. 4: Total Internal Reflection

$\varepsilon_1 = 2\varepsilon_2,\ \mu_1 = \mu_2$ and θ_i: from 0° to 90°

Incident and Transmitted Waves for perpendicular polarization



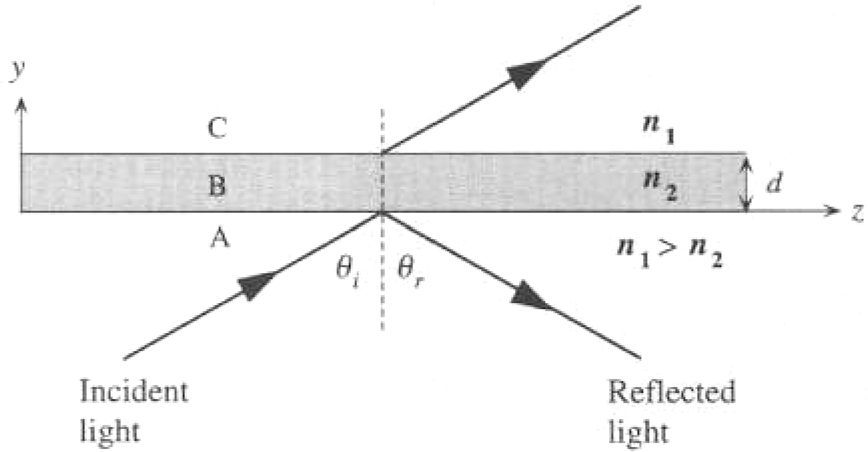
Lect. 4: Total Internal Reflection



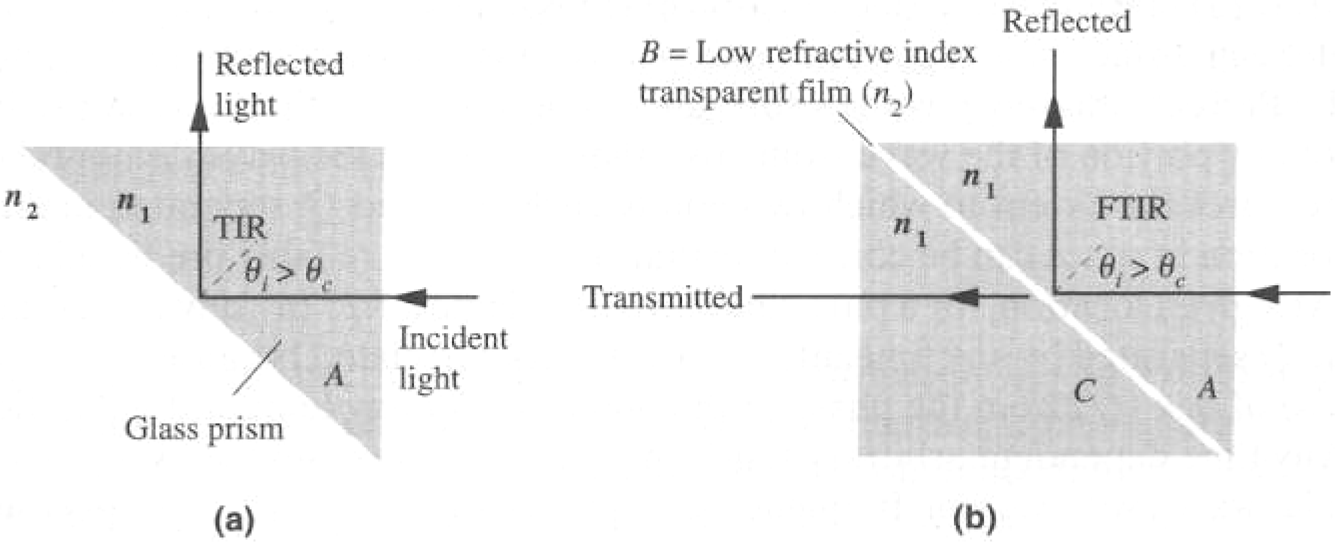
$r = e^{j\phi}$

Goos-Hänchen phase shift

Frustrated Total Internal Reflection



Lect. 4: Total Internal Reflection



Beam Splitter

Exercise Problems:

Prob. 3 (a),(b),(c), Prob. 6,